Electroweak symmetry breaking in TeV-scale string models

Noriaki Kitazawa

Department of Physics, Tokyo Metropolitan University,
Hachioji, Tokyo 192-0397, Japan
e-mail: kitazawa@phys.metro-u.ac.jp

Abstract. We propose a scenario of the electroweak symmetry breaking by one-loop radiative corrections in a class of string models with D3-branes at non-supersymmetric orbifold singularities with the string scale in TeV region. As a test example, we consider a simple model based on a D3-brane at locally $\mathbf{C}^3/\mathbf{Z}_6$ orbifold singularity, and the electroweak Higgs doublet fields are identified with the massless bosonic modes of the open string on that D3-brane. They have Yukawa couplings with three generations of left-handed quarks and right-handed up-type quarks which are identified with the massless fermionic modes of the open string on the D3-brane. We calculate the one-loop correction to the Higgs mass due to the non-supersymmetric string spectrum and interactions, and qualitatively suggest that the negative mass squared can be generated. The problems which must be solved to proceed quantitative calculations are pointed out.

Keywords: string phenomenology, electroweak symmetry breaking
PACS: 11.25.Wx, 12.60.Fr

INTRODUCTION

The dynamics of the electroweak symmetry breaking is still unknown. The standard model has the naturalness or fine-tuning problem, and its minimal supersymmetric extension also requires a certain level of fine-tuning[1]. Although the technicolor dynamics (without supersymmetry) [2, 3] is a candidate of the "inevitable" electroweak symmetry breaking, like the chiral symmetry breaking in QCD, it has some problems to be overcome (for recent works, see refs.[4, 5], for example). The dynamical electroweak symmetry breaking using controllable dynamics of supersymmetric gauge theories is proposed in refs.[6, 7].

In this article we propose an alternative dynamics of "inevitable" electroweak symmetry breaking in string theory without supersymmetry with the string scale in TeV region. This is the idea which has already proposed in ref.[8]. The authors calculated one-loop effective potential for certain scalar fields in a non-supersymmetric brane-anti-brane system ($D9$-$\overline{D5}$ system), and found that these scalar fields can have non-zero vacuum expectation values. These scalar fields correspond to Wilson lines and brane moduli, and the one-loop effective potential can be obtained by a modification of the open string vacuum amplitude at one loop. The vacuum expectation values of these scalar fields break original gauge symmetry $\mathrm{USp}(16)\times\mathrm{USp}(16)$. In this letter we consider the similar phenomena in different systems of D-branes at singularities[9], in which the Higgs doublet fields do not correspond to Wilson lines and brane moduli. The two point function of the Higgs doublet fields, namely the correction to the Higgs masses, have to be directly

calculated to see whether they can have vacuum expectation values or not.

In the next section, we briefly review the system of D3-branes at orbifold singularities. In the third section, we calculate the one-loop two point function of the gauge boson from the open string in ten dimensions. This calculation gives a good guidance to the calculation of the Higgs mass correction. In the last section, we calculate the one-loop correction to the mass of the Higgs doublet field which is realized on the D3-brane at non-supersymmetric locally $\mathbf{C}^3/\mathbf{Z}_6$ orbifold singularity. We suggest that the negative mass squared can be generated. Some concluding comments are also included in this section.

D-BRANES AT SINGULARITIES

We introduce one specific semi-realistic system based on a D3-brane at locally $\mathbf{C}^3/\mathbf{Z}_6$ orbifold singularity. A complete and self-contained introduction to the system of D3- and D7-branes located at singularities, is given in ref.[9].

Consider type IIB theories with six dimensions compactified to orbifolds or orientifolds, and assume that there is a locally $\mathbf{C}^3/\mathbf{Z}_6$ orbifold singularity in the compact six dimensional space. The open string states on the D3-brane at that singularity are modified by the $\mathbf{Z}_6$ projection. If we simply have N coincident D3-brane, the massless states in its four-dimensional world-volume belong to a $\mathscr{N}=4$ supersymmetric $\mathrm{U}(N)$ gauge multiplet. The $\mathbf{Z}_6$ projection breaks the supersymmetry and the gauge symmetry (to the group with the same rank, $\mathrm{U}(N_1)\times\mathrm{U}(n_2)\times\mathrm{U}(n_3)$ with $n_1+n_2+n_3=N$, for example), and some states of the original gauge fields remain the states of gauge fields, some other states correspond to massless matter fields in bi-fundamental representations under the new gauge symmetry, and there are some states which are completely projected out. If the projection keeps four-dimensional $\mathscr{N}=1$ supersymmetry, the resultant numbers of bosonic and fermionic degrees of freedom are equal, and the structure of $\mathscr{N}=1$ supermultiplet remains. There are some consistent ways of projection in which there is no correspondence between bosonic and fermionic states and supersymmetry is completely broken by the spectrum.

In the language of $\mathscr{N}=1$ supermultiplets the original massless field contents on N D3-brane is a $\mathrm{U}(N)$ gauge vector multiplet and three chiral multiplets in the adjoint representation under $\mathrm{U}(N)$. The scalar components of three chiral multiplets can be understood as the position moduli of D3-brane, and they can be understood as the local complexified coordinates of the compact six dimensional space. There is $\mathrm{SU}(4)_R$ global symmetry under which these six bosonic degrees of freedom belong sextet and the four fermionic degrees of freedom, three chiral fermions in three chiral multiplets and gaugino, belong quartet. If the $\mathbf{Z}_6$ transformation is the subgroup of $\mathrm{SU}(4)_R$, the projection is consistent under the time evolution. The $\mathbf{Z}_6$ transformation on the three local complexified coordinates is described by three integers $b_1,b_2,b_3=0,1,\cdots,5$ with the transformation matrix $\mathrm{diag}(e^{2\pi ib_1/6},e^{2\pi ib_2/6},e^{2\pi ib_3/6})$. This is the transformation matrix on a sextet for any possible values of b_i. The $\mathbf{Z}_6$ transformation on four fermionic degrees of freedom is described by four integers $a_1,a_2,a_3,a_4=0,1,\cdots,5$ with $a_1+a_2+a_3+a_4=0$ mod 6 with the transformation matrix $\mathrm{diag}(e^{2\pi ia_1/6},e^{2\pi ia_2/6},e^{2\pi ia_3/6},e^{2\pi ia_3/6})$. Two sets of integers have the relations of $b_1=a_2+a_3$, $b_2=a_3+a_1$ and $b_3=a_1+a_2$. The condi-

tion to have $\mathcal{N}=1$ supersymmetry is $a_4=0$, and in this case $b_r=-a_r$ with $r=1,2,3$. Both three complexified bosonic and three complexified fermionic open string world-sheet fields, which correspond to three local complexified coordinates in compact six dimensional space, transform in the same way as three local complexified coordinates. The transformation of the space-time fermion is realized by the various spin combinations of the vacuum states of Ramond sector.

The open string Chan-Paton factor may transform under $\mathbf{Z}_6$. The transformation is described as

$$|ij\rangle \longrightarrow (\gamma_3)_{ii'}|i'j'\rangle(\gamma_3^{-1})_{j'j}, \tag{1}$$

where $\gamma_3=\mathrm{diag}(I_{n_0}, e^{2\pi i/6}I_{n_1}, \cdots e^{2\pi i\cdot 5/6}I_{n_5})$ with $n_0+n_1+\cdots+n_5=N$ and I_n is $n\times n$ unit matrix.

The massless open string states, which are singlet under $\mathbf{Z}_6$ transformation, correspond to the massless fields. In addition to the gauge bosons of the gauge symmetry of $\mathrm{U}(n_0)\times\mathrm{U}(n_1)\times\cdots\times\mathrm{U}(n_5)$, we have massless matter fields in bi-fundamental representation:

$$\text{complex scalars} \qquad \sum_{r=1}^{3}\sum_{i=0}^{5}(n_i, \bar{n}_{i-b_r}), \tag{2}$$

$$\text{Weyl fermions} \qquad \sum_{\alpha=1}^{3}\sum_{i=0}^{5}(n_i, \bar{n}_{i+a_\alpha}), \tag{3}$$

where n_i and $\bar{n_i}$ mean fundamental and anti-fundamental representation of $\mathrm{U}(n_i)$, respectively.

We proceed to much more concrete model. We take $N=6$ and $n_0=1$, $n_1=3$, $n_2=2$ and $n_3=n_4=n_5=0$. For $\mathbf{Z}_6$ projection, we take $b_1=b_2=b_3=2$, $a_1=a_2=a_3=1$ and $a_4=-3$. This set up gives non-supersymmetric spectrum. We have gauge symmetry of $\mathrm{U}(3)\times\mathrm{U}(2)\times\mathrm{U}(1)$ and massless matter fields

$$\text{Higgs doublet fields:} \quad H_r \qquad 3\times(1,2,-1), \tag{4}$$

$$\text{left-handed quarks:} \quad q_{Lr} \qquad 3\times(3,2^*,0), \tag{5}$$

$$\text{right-handed quarks:} \quad {u^c}_{Lr} \qquad 3\times(3^*,1,+1), \tag{6}$$

where we omit to describe the charges of $\mathrm{U}(1)$ factors of $\mathrm{U}(3)$ and $\mathrm{U}(2)$. There are Yukawa couplings among these fields which are obtained by the $\mathbf{Z}_6$ projection of the interactions due to the superpotential in the original $\mathcal{N}=4$ supersymmetric theory. The Yukawa coupling constants are equal to the gauge coupling constant. Note that all the gauge coupling constants of $\mathrm{U}(3)\times\mathrm{U}(2)\times\mathrm{U}(1)$ are equal at tree level.

There are four point Higgs self-couplings which are remnants of the D-term scalar potential in the original supersymmetric theory.

$$V=\frac{g^2}{4}\sum_{r,s=1,2,3}\left((H_r^\dagger H_s)(H_s^\dagger H_r)+(H_r^\dagger H_r)(H_s^\dagger H_s)\right), \tag{7}$$

where g is the gauge coupling constant of D3-brane. There are no flat directions on the vacuum expectation values of Higgs doublet fields. This means that Higgs doublet fields are not D-brane moduli.

This is not the complete system. We have to consider the R-R (Ramond-Ramond) tadpole cancellation to make a consistent string theory. R-R tadpole cancellation conditions contain chiral anomaly cancellation conditions. We have to consider both twisted and untwisted tadpoles. Twisted tadpoles can be cancelled out by introducing appropriate D7-branes. Inclusion of D7-branes means introduction of additional gauge symmetry and massless matter fields which ensure the chiral anomaly cancellation about the gauge symmetry on D3-brane. To consider the untwisted tadpole cancellation we have to specify a concrete compactification space. Since the construction of the concrete realistic models is not the issue of this letter, and the existence of the untwisted tadpole does not affect our forthcoming discussions, we do not discuss the untwisted tadpole cancellation.

ONE-LOOP TWO POINT FUNCTION IN TEN DIMENSIONS

We consider two point functions of gauge bosons in the zero momentum limit, namely the mass corrections to gauge bosons. Although we know the result: it should vanish because of the gauge invariance, it is instructive to calculate the mass correction to the Higgs doublet fields.

Before going to the calculation in string theory, it is worth to mention the calculation in four dimensional $\mathscr{N}=1$ supersymmetric U(1) gauge field theory. The quadratically divergent contributions come from two boson loop diagrams and one fermion loop diagram:

$$\Pi^{\mu\nu}_{\text{boson}} = (1-2)\times\eta^{\mu\nu}\int\frac{d^4p}{(2\pi)^4 i}\,\frac{1}{p^2}, \tag{8}$$

$$\Pi^{\mu\nu}_{\text{fermion}} = \eta^{\mu\nu}\int\frac{d^4p}{(2\pi)^4 i}\,\frac{1}{p^2}, \tag{9}$$

where $\eta^{\mu\nu}=\text{diag}(-1,1,1,1)$. These are the contributions of one chiral multiplet. We see the cancellation of these two corrections due to supersymmetry. If we take Euclidean momentum cutoff regularization, boson and fermion give positive and negative correction to the mass squared of the gauge boson, respectively. If we take the gauge invariant regularization, dimensional regularization, for example, the corrections vanish individually.

We use the techniques for one-loop calculation in open string theory, which is described in the text book of ref.[10], because it has good correspondence with the one-loop calculation in field theory. The one-loop contributions of the bosonic states and fermionic states to the two point function of the massless vector mode of the open string (gauge field) are respectively described by

$$A^{\text{NS}} = \int\frac{d^{10}p}{(2\pi)^{10} i}\text{tr}\left(\Delta V(1)\Delta V(1)P_{\text{GSO}}\right), \tag{10}$$

$$A^{\text{R}} = -\int\frac{d^{10}p}{(2\pi)^{10} i}\text{tr}\left(SW(1)SW(1)P_{\text{GSO}}\right), \tag{11}$$

where NS and R mean Neveu-Schwarz and Ramond sector, respectively, $P_{\rm GSO}$ is the Gliozzi-Scherk-Olive projection operator, propagators are defined as

$$\Delta \equiv \int_0^1 x^{L_0-1}dx, \tag{12}$$
$$S \equiv iG_0\Delta, \tag{13}$$

and vertex operators are defined as

$$W(1) = g_O e_\mu \psi^\mu e^{ik\cdot X}, \tag{14}$$
$$\begin{aligned} V(1) &\equiv \{G_0, W(1)\} \\ &= \frac{g_O}{\sqrt{2\alpha'}} e_\mu \left(i\dot{X}^\mu + 2\alpha'(k\cdot\psi)\psi^\mu\right) e^{ik\cdot X} \end{aligned} \tag{15}$$

with open string coupling constant g_O and polarization vector e_μ. The argument 1 of vertex operators means the complex valuable $z = \exp(-i(\sigma_1 + i\sigma_2))$ with the value of the Euclidean world-sheet coordinates $\sigma_1 = \sigma_2 = 0$. Taking the value of $\sigma_1 = 0$ means that we are considering planner diagrams: both two vertex operators are attached to one of two boundaries of annulus (or cylinder). We do not discuss the non-planner diagram, because it is not important to the calculation of the mass correction to the Higgs doublet fields in the next section. The dot operation in eq.(15) means the differentiation by σ_2.

We evaluate only the leading terms in the internal momentum integration of eqs.(10) and (11). These are dominant contributions because of the following reasons. The open string one-loop calculation is essentially to count the number of possible states in the loop with weight $\exp(-2\pi t L_0)$ and to integrate over the cylinder modulus $0 \le t < \infty$ ($2\pi t$ is the circumference of the cylinder). The integrant of the modulus integration mainly has value in the regions of small t because of the exponential weight. The leading internal momentum integration gives an enhancement factor for the small t region, which is absent in the sub-leading momentum integration. Therefore the leading term in the internal momentum integration can be considered as the dominant contribution. In four dimensional field theory, this corresponds to evaluate only the quadratically divergent terms like in eqs.(8) and (9).

The results of the calculation is the following.

$$A^{\rm NS} \simeq \frac{1}{2}\frac{g_O^2}{\alpha'^5} e^\mu e_\mu \int_0^1 \frac{d\rho}{\rho}\left(\frac{\pi}{-\ln\rho}\right)^5 \frac{1}{\eta(it)^8}\left(\left(\frac{\theta_3(it)}{\eta(it)}\right)^4 - \left(\frac{\theta_4(it)}{\eta(it)}\right)^4\right), \tag{16}$$
$$A^{\rm R} \simeq -\frac{1}{2}\frac{g_O^2}{\alpha'^5} e^\mu e_\mu \int_0^1 \frac{d\rho}{\rho}\left(\frac{\pi}{-\ln\rho}\right)^5 \frac{1}{\eta(it)^8}\left(\frac{\theta_2(it)}{\eta(it)}\right)^4, \tag{17}$$

where $\rho = \exp(-2\pi t)$, η is Dedekind eta function and θ_2, θ_3 and θ_4 are Jacobi theta functions. Here we have taken the limit of zero external momentum. The total correction vanishes because of the relation $(\theta_3)^4 - (\theta_4)^4 - (\theta_2)^4 = 0$, which means the balance of the number of bosonic and fermionic states, namely supersymmetry. More rigorous calculation requires to include the contribution from non-planner and nonorientable diagrams in type I theory with the care of Chan-Paton factor as well as the sub-leading

contribution of the internal momentum integration. However, this level of calculation is enough to give a guide to the calculation of the one-loop mass correction of the Higgs doublet fields. In case of no supersymmetry in the spectrum, boson and fermion contributions are not necessary balanced. If the boson contribution is dominant, the correction to the mass squared is positive, and if the fermion contribution is dominant, the correction to the mass squared is negative.

ONE-LOOP CORRECTION TO THE HIGGS MASS

The calculation is very similar to that in the previous section. The essential difference is the boundary condition of the world-sheet fields as well as $\mathbf{Z}_6$ projection. Among the ten world-sheet boson and ten world-sheet fermion fields, the fields corresponding to the parallel direction to D3-brane follow Neumann boundary condition, and others, namely the fields corresponding to the transverse direction to D3-brane follow Dirichlet boundary condition for both edges of the open string. One important fact is that there is no string center of mass momentum of the transverse direction to D3-brane.

The amplitudes to be calculated are as follows.

$$A^{\rm NS}_{\rm Higgs} = \int \frac{d^4p}{(2\pi)^4 i} {\rm tr}\left(\Delta V(1)^{(-)} \Delta V(1)^{(+)} P_{\rm GSO} P_{\mathbf{Z}_6}\right), \tag{18}$$

$$A^{\rm R}_{\rm Higgs} = -\int \frac{d^4p}{(2\pi)^4 i} {\rm tr}\left(SW(1)^{(-)} SW(1)^{(+)} P_{\rm GSO} P_{\mathbf{Z}_6}\right), \tag{19}$$

where $P_{\mathbf{Z}_6}$ is the Z_6 projection operator, vertex operators are defined as

$$W(1)^{(+)} = g_O u^{i_1}{}_{i_2} \psi^{(+)} e^{ik\cdot X}, \tag{20}$$

$$\begin{aligned} V(1)^{(+)} &\equiv \{G_0, W(1)^{(+)}\} \\ &= \frac{g_O}{\sqrt{2\alpha'}} u^{i_1}{}_{i_2} \left(i\dot{X}^{(+)} + 2\alpha'(k\cdot\psi)\psi^{(+)}\right) e^{ik\cdot X} \end{aligned} \tag{21}$$

with

$$X^{(\pm)} = \frac{1}{\sqrt{2}}\left(X^4 \pm iX^5\right), \qquad \psi^{(\pm)} = \frac{1}{\sqrt{2}}\left(\psi^4 \pm i\psi^5\right) \tag{22}$$

for one of three Higgs doublet states, and the momentum k_μ can have non-zero value only for $\mu = 0,1,2,3$. The vertex operators $W(1)^{(-)}$ and $V(1)^{(-)}$ are Hermite conjugates of $W(1)^{(+)}$ and $V(1)^{(+)}$, respectively. The indexes i_1 and i_2 of the factor $u^{i_1}{}_{i_2}$ denote the Chan-Paton factor of $\mathrm{U}(1)$ and $\mathrm{U}(2)$, respectively. By considering the flow of the Chan-Paton charge, it is easily understood that only the planner diagram contributes.

Now, we calculate the leading terms of internal momentum integration, which are dominant contributions. We find that there is no leading term in Neveu-Schwarz amplitude (boson loop), because $X(1)^{(\pm)}$ has no zero modes due to Dirichlet boundary condition. Therefore, we may only calculate Ramond amplitude (fermion loop), and make sure whether it vanishes or not. If it is not zero, we have a negative correction to the Higgs mass squared, and the electroweak symmetry breaking by the Higgs vacuum

expectation value through the one-loop quantum effect is expected. With a special care of $\mathbf{Z}_6$ projection, we obtain the following result.

$$
\begin{aligned}
A^{\rm R}_{\rm Higgs} \;\simeq\; & -3 \cdot \frac{1}{2} \frac{g_O^2}{\alpha'^2} u^{\dagger i_2}{}_{i_1} u^{i_1}{}_{i_2} \int_0^1 \frac{d\rho}{\rho} \left(\frac{\pi}{-\ln \rho} \right)^2 \qquad (23) \\
& \times \frac{1}{6} \sum_{\gamma=0}^{5} 16 \prod_{m=1}^{\infty} \left(\frac{1+\rho^m}{1-\rho^m} \right)^2 \left(\frac{1+(e^{2\pi i/3})^\gamma \rho^m}{1-(e^{2\pi i/3})^\gamma \rho^m} \right)^3 \left(\frac{1+(e^{-2\pi i/3})^\gamma \rho^m}{1-(e^{-2\pi i/3})^\gamma \rho^m} \right)^3 ,
\end{aligned}
$$

where the first factor 3 is the Chan-Paton factor. The only $U(3)$ charged stats, including massless left-handed quarks and right-handed up-type quarks, contribute the loop. Although certainly this amplitude is not zero, we have to consider more model dependent contributions due to the twisted R-R tadpole cancellations. That is a required contribution which completely cancels the one-loop correction in case with supersymmetry.

Here, to cancel the twisted R-R tadpoles, we introduce 36 D7-brane whose world-volume is our four dimensional space-time and second and third complex planes in six dimensional compact space. We take the $\mathbf{Z}_6$ action to the Chan-Paton factor of this D7-brane as $\gamma_7 = {\rm diag}(I_{u_0}, e^{2\pi i/6} I_{u_1}, \cdots e^{2\pi i \cdot 5/6} I_{u_5})$ with $u_0 = 6$, $u_1 = 0$, $u_2 = 3$ and $u_3 = u_4 = u_5 = 9$. This D7-brane gives new gauge symmetries ${\rm U}(6) \times {\rm U}(3) \times {\rm U}(9)_1 \times {\rm U}(9)_2 \times {\rm U}(9)_3$ with very small gauge coupling constants, since we take the string scale in TeV range. This symmetries emerge as global symmetries at low energies. We have new massless and massive states form the open string with one edge on D3-brane and another edge on D7-brane. Although we do not explain detailed spectrum here, since the concrete model building is not the aim of this letter, we would like to stress that there are no massless fermion states which have Yukawa couplings with Higgs doublet fields. This is an important difference from the case with supersymmetry. The leading one-loop correction to the Higgs mass squared from this open string is obtained as follows.

$$
\begin{aligned}
A'^{\rm R}_{\rm Higgs} \;\simeq\; & -9 \cdot \frac{1}{2} \frac{g_O^2}{\alpha'^2} u^{\dagger i_2}{}_{i_1} u^{i_1}{}_{i_2} \int_0^1 \frac{d\rho}{\rho} \left(\frac{\pi}{-\ln \rho} \right)^2 \cdot \frac{1}{6} \sum_{\gamma=0}^{5} \left((e^{2\pi i/3})^\gamma + (e^{-2\pi i/3})^\gamma \right) \\
& \times 16 \prod_{m=1}^{\infty} \left(\frac{1+\rho^m}{1-\rho^m} \right)^2 \left(\frac{1+(e^{2\pi i/3})^\gamma \rho^m}{1-(e^{2\pi i/3})^\gamma \rho^m} \right) \left(\frac{1+(e^{-2\pi i/3})^\gamma \rho^m}{1-(e^{-2\pi i/3})^\gamma \rho^m} \right) \\
& \times \left(\frac{1+(e^{2\pi i/3})^\gamma \rho^{m-1/2}}{1-(e^{2\pi i/3})^\gamma \rho^{m-1/2}} \right)^2 \left(\frac{1+(e^{-2\pi i/3})^\gamma \rho^{m-1/2}}{1-(e^{-2\pi i/3})^\gamma \rho^{m-1/2}} \right)^2 , \qquad (24)
\end{aligned}
$$

where the first factor 9 is the Chan-Paton factor, and the factor $((e^{2\pi i/3})^\gamma + (e^{-2\pi i/3})^\gamma)$ means the non-trivial transformation of Ramond sector vacuum under the $\mathbf{Z}_6$ action, which is related with the fact that no massless fermion states have Yukawa couplings with the Higgs doublet. Only the massive states with ${\rm U}(9)_1$ or ${\rm U}(9)_3$ charges contribute. The last two factors in the infinite product are the realizations of the Dirichlet-Neumann boundary condition of the open string in second and third complex planes in compact six dimensional space. The contribution of eq.(24) certainly does not cancel out the contribution of eq.(23), though the divergent contributions from twisted R-R tadpoles

are cancelled out. The finite contribution of eq.(24) should be negative, since it is also due to the fermion one loop. We suggest that the negative mass squared of the Higgs doublet field can be generated, and the electroweak symmetry breaking can be expected, in this class of non-supersymmetric models with D-branes at singularities.

There are some concluding comments in order.

There may still exist a divergence in the total correction of eq.(23) plus eq.(24) due to the uncanceled twisted NS-NS (Neveu-Schwarz-Neveu-Schwarz) tadpoles. Since there is no supersymmetry, R-R tadpole cancellation does not necessary result NS-NS tadpole cancellation. The existence of NS-NS tadpole means that some redefinitions of backgrounds are required [11, 12, 13, 14]. It is probable that twisted moduli obtain vacuum expectation values as a result of the background redefinition by the uncanceled twisted NS-NS tadpoles. The vacuum expectation values of twisted moduli give some modification of Higgs potential of eq.(7) due to the emergence of the Fayet-Iliopoulos terms for anomalous $\mathrm{U}(1)$ gauge symmetries in the original supersymmetric theory [15, 16]. This effect by itself can give vacuum expectation value to Higgs doublet fields even without negative mass squared at one loop. The vacuum expectation values of twisted moduli may also result blowing-up the orbifold singularity (see for example [17]), and the actual geometrical D3-brane reconfiguration by the vacuum expectation value of Higgs doublet fields may be understood at this blown-up orbifold singularity.

There is another possibility that the present non-supersymmetric $\mathbf{C}^3/\mathbf{Z}_6$ orbifold singularity is unstable and decays to some non-singular point. In other words, there might be tachyon modes in twisted NS-NS closed string sector localized at the non-supersymmetric singularity, which means the instability of the singularity[18]. The non-supersymmetric orbifold singularities suffer from this phenomenon in general. We need to find stable non-supersymmetric singularities without tachyons or non-supersymmetric singularities with very long life time.

There are three Higgs doublet fields, H_1, H_2 and H_3, in our $\mathbf{Z}_6$ model, and our calculations concern one of them, H_1. The contribution of eq.(23) is applicable for all three Higgs doublet fields, but the contribution, which is related with the R-R tadpole cancellations, may be different depending on models. In our model with one D7-brane, the result of eq.(24) is only for H_1, and H_2 and H_3 will obtain a different result. This kind of asymmetric configuration of D7-branes for twisted R-R tadpole cancellation gives asymmetric corrections to Higgs doublet fields. It may be possible that only one Higgs doublet field, which is a linear combination of Higgs doublet fields, has vacuum expectation value, and others are heavy. This is a phenomenologically preferable situation, since the existence of many Higgs doublet fields with vacuum expectation values causes a problem, flavor-changing neutral current problem, in general. Such a Higgs doublet field may have non-trivial Yukawa couplings for the hierarchical masses and flavor mixings, though the original Higgs doublet fields usually have trivial Yukawa couplings.

Inclusion of sub-leading term is important for more quantitative and detailed analysis. Modern techniques, like path integral formalism, might be better for this aim.

ACKNOWLEDGMENT

I would like to thank O. Yasuda for useful comments.

REFERENCES

1. G. L. Kane, J. Lykken, B. D. Nelson and L. T. Wang, Phys. Lett. B **551** (2003) 146 [arXiv:hep-ph/0207168].
2. S. Weinberg, Phys. Rev. D **19** (1979) 1277.
3. L. Susskind, Phys. Rev. D **20** (1979) 2619.
4. D. K. Hong, S. D. H. Hsu and F. Sannino, Phys. Lett. B **597** (2004) 89 [arXiv:hep-ph/0406200].
5. N. D. Christensen and R. Shrock, arXiv:hep-ph/0509109.
6. R. Harnik, G. D. Kribs, D. T. Larson and H. Murayama, D **70** (2004) 015002 [arXiv:hep-ph/0311349].
7. N. Haba and N. Okada, arXiv:hep-ph/0409113.
8. I. Antoniadis, K. Benakli and M. Quiros, Nucl. Phys. B **583** (2000) 35 [arXiv:hep-ph/0004091].
9. G. Aldazabal, L. E. Ibanez, F. Quevedo and A. M. Uranga, JHEP **0008** (2000) 002 [arXiv:hep-th/0005067].
10. M. B. Green, J. H. Schwarz and E. Witten, “Superstring Theory,” Vol. 1 and Vol. 2, Cambridge Univ. Pr. (1987).
11. W. Fischler and L. Susskind, Phys. Lett. B **171** (1986) 383.
12. W. Fischler and L. Susskind, Phys. Lett. B **173** (1986) 262.
13. S. R. Das and S. J. Rey, Phys. Lett. B **186** (1987) 328.
14. E. Dudas, G. Pradisi, M. Nicolosi and A. Sagnotti, Nucl. Phys. B **708** (2005) 3 [arXiv:hep-th/0410101].
15. M. R. Douglas and G. W. Moore, arXiv:hep-th/9603167.
16. M. R. Douglas, B. R. Greene and D. R. Morrison, Nucl. Phys. B **506** (1997) 84 [arXiv:hep-th/9704151].
17. M. Cvetic, L. L. Everett, P. Langacker and J. Wang, JHEP **9904** (1999) 020 [arXiv:hep-th/9903051].
18. A. Adams, J. Polchinski and E. Silverstein, JHEP **0110** (2001) 029 [arXiv:hep-th/0108075].